Name: ____________________________

Math 10560, Final Exam:
May 7, 2014

Instructor: _________________________

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- No **calculators** should be used during the exam.
- **Turn off and put away all cell-phones** and similar electronic devices.
- **Head phones** are not allowed.
- **Put away all notes and formula sheets** where they cannot be viewed.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 15 pages of the test.
- Hand in the entire exam.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)
13.	(a)	(b)	(c)	(d)	(e)
14.	(a)	(b)	(c)	(d)	(e)
15.	(a)	(b)	(c)	(d)	(e)
16.	(a)	(b)	(c)	(d)	(e)
17.	(a)	(b)	(c)	(d)	(e)
18.	(a)	(b)	(c)	(d)	(e)
19.	(a)	(b)	(c)	(d)	(e)
20.	(a)	(b)	(c)	(d)	(e)
21.	(a)	(b)	(c)	(d)	(e)
22.	(a)	(b)	(c)	(d)	(e)
23.	(a)	(b)	(c)	(d)	(e)
24.	(a)	(b)	(c)	(d)	(e)
25.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice

1.(6 pts.) The function $f(x) = \frac{1}{2}\ln(1+x) + e^x$ is one-to-one (you do not need to check this). Find the equation of the tangent line to the inverse function $f^{-1}(x)$ at the point $(1, f^{-1}(1))$.

(a) $y = \dfrac{2}{3}x - \dfrac{2}{3}$ (b) $y = \dfrac{2}{3}x - 1$ (c) $y = 2x - 3$

(d) $y = \dfrac{3}{2}x + 1$ (e) $y = \dfrac{1}{2}x + \dfrac{1}{2}$

2.(6 pts.) Solve for x in the following equation:

$$\ln(e^x - 1) + \ln(e^x + 1) = 2.$$

(a) $-\dfrac{3}{3}$ (b) 1 (c) $\dfrac{e^2+1}{2}$

(d) $\dfrac{\ln(e^2+1)}{2}$ (e) $\dfrac{3}{2}$

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3.(6 pts.) Evaluate the derivative of

$$f(x) = \frac{\ln(\arcsin(x^2))}{2}.$$

(Recall: $\arcsin y = \sin^{-1} y$.)

(a) $\dfrac{1}{2\sqrt{1-x^4}\ (\arcsin(x^2))}$

(b) $\dfrac{1}{(1+[\arcsin(x^2)]^2)}$

(c) $\dfrac{x}{\sqrt{1-x^4}\ (\arcsin(x^2))}$

(d) $\dfrac{x}{(1+x^4)\ (\arcsin(x^2))}$

(e) $\dfrac{-x}{\sin^2(x^2)(\arcsin(x^2))}$

4.(6 pts.) Evaluate the derivative of the function

$$f(x) = (\sin x)^{1/x^2}.$$

(a) $\dfrac{(\sin x)^{(1/x^2)-1}\cos x}{x^2}$

(b) $(\sin x)^{1/x^2}\left[x^2\cos x + 2x\ln(\sin x)\right]$

(c) $(\sin x)^{1/x^2}\left[\dfrac{x^2\cos x - 2x\ln(\sin x)}{x^4}\right]$

(d) $(\sin x)^{1/x^2}\left[\dfrac{x^2\cot x - 2x\ln(\sin x)}{x^4}\right]$

(e) $\dfrac{x^2\cot x - 2x\ln(\sin x)}{x^4}$

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5.(6 pts.) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} x \cos x dx.$

(a) $\dfrac{\pi}{2}+1$ (b) $\dfrac{\pi}{4}$ (c) 0 (d) $\dfrac{\pi}{2}-1$ (e) -1

6.(6 pts.) Which of the definite integrals shown below is equal to the definite integral

$$\int_0^2 \frac{x^2}{\sqrt{x^2+4}}\, dx?$$

(**Note:** A trigonometric substitution might help.)

(a) $\displaystyle\int_0^{\frac{\pi}{4}} 4\tan^2\theta \sec\theta \, d\theta$

(b) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{2\tan^2\theta}{\sec\theta}\, d\theta$

(c) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{2\tan^2\theta}{\sec\theta}\, d\theta$

(d) $\displaystyle\int_0^{\frac{\pi}{2}} 4\tan^2\theta \sec\theta \, d\theta$

(e) $\displaystyle\int_0^{\frac{\pi}{2}} 4\tan^2\theta \, d\theta$

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7.(6 pts.) Evaluate the integral

$$\int_1^e \frac{x^2+x+1}{x(x^2+1)}\,dx.$$

(Recall: $\arctan x = \tan^{-1} x$ and $\arcsin x = \sin^{-1} x$.)

(a) $1+\arctan(e)$

(b) $1-\frac{\pi}{2}+\arcsin(e)$

(c) $1-\frac{\pi}{4}+\arctan(e)$

(d) $1-\arctan(e)$

(e) $1-\arcsin(e)$

8.(6 pts.) Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \sin^{100} x \cos^3 x\,dx.$$

(a) $\frac{1}{101}-\frac{1}{103}$

(b) $\frac{\pi^{100}}{2^{100}(100)}-\frac{\pi^{102}}{2^{102}(102)}$

(c) $\frac{1}{100}-\frac{1}{102}$

(d) $\frac{\pi^{101}}{2^{101}(101)}-\frac{\pi^{103}}{2^{103}(103)}$

(e) $\frac{1}{101}$

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9.(6 pts.) Use Simpson's rule with $n = 6$ to approximate the integral

$$\int_0^3 f(x)dx,$$

where a table of values of $f(x)$ is given below.

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	1	0.5	0.25	0.25	1	1.5	-0.5

Note: The formula sheet may help.

(a) $\frac{13}{3}$ (b) 12 (c) 2 (d) $\frac{13}{6}$ (e) 4

10.(6 pts.)Determine whether the following integral converges or diverges. If it converges, evaluate it.

$$\int_0^\infty \frac{e^x}{1+e^{2x}}dx.$$

(a) $\frac{\pi}{2}$

(b) The integral diverges.

(c) 1

(d) $\frac{\pi}{4}$

(e) 2

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11.(6 pts.)The length of the curve $y = \dfrac{x^3}{6} + \dfrac{1}{2x}, \quad \frac{1}{2} \leq x \leq 1$, is given by:

(a) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{1 + (x^2 + x^{-2})^2}dx$

(b) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{1 + (x + x^{-1})^2}dx$

(c) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} (x + x^{-1})dx$

(d) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} (x^2 + x^{-2})dx$

(e) $\dfrac{1}{2}\displaystyle\int_{1/2}^{1} \sqrt{(x^2 + x^{-2})}dx$

12.(6 pts.)The solution to the initial value problem

$$x\frac{dy}{dx} + 2y = e^{x^2} \qquad y(1) = 0$$

is

(a) $y = \dfrac{e^{x^2} - e}{2x^2}$

(b) $y = \dfrac{e^{x} - e}{2x^2}$

(c) $y = \dfrac{e^{x} - e}{2x}$

(d) $y = \dfrac{e^{x^2} - e}{2x}$

(e) $y = xe^x - e$

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13.(6 pts.)The solution to the initial value problem

$$y' = x\cos^2 y \qquad\qquad y(2) = 0$$

satisfies the implicit equation

(a) $\cos y = \frac{x^2}{2} - 2$

(b) $\ln|\sec(y) + \tan(y)| = \frac{x^2}{2} - 2$

(c) $\tan y = \frac{x^2}{2} - 2$

(d) $\sin(2y) = 4x - 8$

(e) $\arctan y = \frac{x^2}{2} - 2$

14.(6 pts.)Consider the initial value problem

$$\begin{cases} y' &= \sin[\pi(x+y)] \\ y(0) &= 0. \end{cases}$$

Use Euler's method with two steps of step size 0.5 to find an approximate value of $y(1)$.
Note: The formula sheet may help.

(a) 0 (b) 0.5 (c) -1 (d) 1 (e) -0.5

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15.(6 pts.) Consider the following **sequences**:

$$(I)\quad \left\{(-1)^n \frac{n-1}{2n^2+1}\right\}_{n=1}^{\infty} \qquad (II)\quad \left\{\frac{n^2-1}{\ln(n)}\right\}_{n=1}^{\infty} \qquad (III)\quad \left\{3^{1/n}\right\}_{n=1}^{\infty}$$

Which of the following statements is true?

(a) All three sequences diverge.

(b) Sequence III diverges and sequences I and II converge.

(c) Sequence II diverges and sequences I and III converge.

(d) All three sequences converge.

(e) Sequence I diverges and sequences II and III converge.

16.(6 pts.) Find $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{3^{n-1}}$.

(a) $\frac{3}{2}$ (b) $-\frac{9}{2}$ (c) 2 (d) 3 (e) $\frac{15}{2}$

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17.(6 pts.) Consider the following series

$$(I)\quad \sum_{n=3}^{\infty} \frac{(-1)^n}{\ln(n)} \qquad (II)\quad \sum_{n=2}^{\infty} \frac{2n^2}{n^4+1} \qquad (III)\quad \sum_{n=1}^{\infty} \frac{n!}{3^n}$$

Which of the following statements is true?

(a) Only III converges

(b) Only I and II converge

(c) Only II and III converge

(d) All three diverge

(e) All three converge

18.(6 pts.) Consider the following series

$$(I)\quad \sum_{n=3}^{\infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}} \qquad (II)\quad \sum_{n=3}^{\infty} \frac{(-1)^n}{\sqrt[3]{n-1}}.$$

Which of the following statements is true?

(a) (I) is divergent and (II) is conditionally convergent.

(b) (I) is absolutely convergent and (II) diverges.

(c) (I) and (II) are both conditionally convergent.

(d) (I) is absolutely convergent and (II) is conditionally convergent.

(e) (I) and (II) both diverge.

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19.(6 pts.) Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{5^n(n+1)}.$$

(a) $(-5, 5]$ (b) $(-\frac{1}{5}, \frac{1}{5}]$ (c) $[-5, 5)$ (d) $(-1, 1]$ (e) $[-\frac{1}{5}, \frac{1}{5})$

20.(6 pts.) Find a power series representation for the the function $f(x) = \ln(1 + x^3)$ on the interval $-1 < x < 1$.

Hint: $\frac{d}{dx}\left(\ln(1+x^3)\right) = \frac{3x^2}{1+x^3}$.

(a) $\sum_{n=0}^{\infty} \frac{x^{n+3}}{n+4}$ (b) $\sum_{n=0}^{\infty} (-1)^n x^{3n}$ (c) $\sum_{n=0}^{\infty} x^{3n}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n 3x^{3n+1}}{3n+1}$ (e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+3}}{n+1}$

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21.(6 pts.) Which of the power series given below is the McLaurin series (i.e. Taylor series at $a = 0$) of the function

$$f(x) = xe^{x^3}?$$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{3n+1}}{(3n)!}$.

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(2n+1)!}$.

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{3n+1}}{(n)!}$.

22.(6 pts.) Which of the polynomials shown below is the third Taylor polynomial of $f(x) = \dfrac{1}{(1-x)^2}$ at $a = -1$?

(a) $1 + 2x + 3x^2 + 4x^3$

(b) $\dfrac{1}{2^2} + \dfrac{2}{2^3}x + \dfrac{3}{2^4}x^2 + \dfrac{4}{2^5}x^3$

(c) $\dfrac{1}{2^2} + \dfrac{2!}{2^3}(x+1) + \dfrac{3!}{2^4}(x+1)^2 + \dfrac{4!}{2^5}(x+1)^3$

(d) $\dfrac{1}{2^2} + \dfrac{2}{2^3}(x+1) + \dfrac{3}{2^4}(x+1)^2 + \dfrac{4}{2^5}(x+1)^3$

(e) $1 + (x+1) + (x+1)^2 + (x+1)^3$

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23.(6 pts.) Find the length of the parameterized curve

$$x = \cos^2 t, \qquad y = \sin^2 t, \quad 0 \le t \le \frac{\pi}{2}.$$

(a) $\frac{1}{2}$ (b) $\sqrt{2}$ (c) 1 (d) $\frac{\pi}{2}$ (e) $\frac{1}{2\sqrt{2}}$

24.(6 pts.) The point $\left(2, \frac{7\pi}{3}\right)$ in polar coordinates corresponds to which point below in Cartesian coordinates?

(a) $(1, \sqrt{3})$

(b) $(-1, \sqrt{3})$

(c) $(\sqrt{3}, 1)$

(d) $(-\sqrt{3}, 1)$

(e) Since $\frac{7\pi}{3} > 2\pi$, there is no such point

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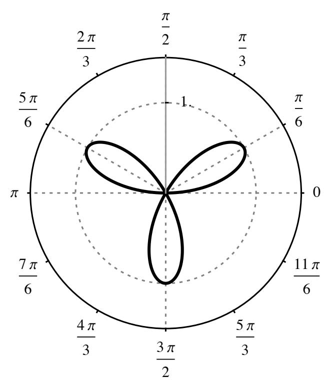
25.(6 pts.) Find the area of the region enclosed by the polar curve

$$r = \sin(3\theta), \quad 0 \leq \theta \leq \pi.$$

(**Note:** The formula sheet may help here.)



(a) 3 (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{2}$ (e) π

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The following is the list of useful trigonometric formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2\sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$

$$\int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta d\theta = \ln|\csc\theta - \cot\theta| + C$$

$$\int \csc^2\theta d\theta = -\cot x + C$$

Trapezoidal Rule If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx \approx T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + +2f(x_{n-1}) + f(x_n))$$

Error Bounds If $|f''(x)| \leq K$ for $a \leq x \leq b$. Let E_T denote the error for the trapezoidal approximation then $|E_T| \leq \frac{K(b-a)^3}{12n^2}$

Simpson's rule If f is integrable on $[a, b]$, then

$$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}(f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n))$$

Error Bound for Simpson's Rule Suppose that $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$. If E_S is the error involved in using Simpson's Rule, then $|E_S| \leq \frac{K(b-a)^5}{180n^4}$

Euler's Method with step size h: $y_i = y_{i-1} + hF(x_{i-1}, y_{i-1})$, $\quad x_i = x_0 + ih$.